

Analyzing the Standards

# **5TH GRADE MATHEMATICS**

### Standards for Mathematical Practice

**5.MP** Display perseverance and patience in problem-solving. Demonstrate skills and strategies needed to succeed in mathematics, including critical thinking, reasoning, and effective collaboration and expression. Seek help and apply feedback. Set and monitor goals.

1 Make sense of problems and persevere in solving them	<b>2</b> Reason abstractly and quantitatively	<b>3</b> Construct viable arguments/critique the reasoning of others	4 Model with mathematics
5 Use appropriate tools strategically	6 Attend to precision	7 Look for and make use of structure	<b>8</b> Look for and express regularity in repeated reasoning

### Big Idea: Numerical Reasoning (NR)

place value, multiplying by powers of 10, multiplication and division of multi-digit numbers, fractions, decimal numbers, numerical expressions

5MA.A.1 use place value understanding to solve real-life, mathematical problems (5.NR.1)
1.a explain that in a multi-digit number, a digit in one place represents 10 times as much as it represents in the place to its right (up to 100 times greater) and 1/10 of what it represents in the place to its left (up to 1/1000 of the value)
1.b explain patterns in the placement of digits when multiplied or divided by a power of 10; use whole-number exponents to denote powers of 10, up to 10<sup>3</sup>

Overview



Analyzing the Standards

### What does this standard mean that a student must know, understand, or be able to do?

### **Description of the Standard**

When learning this standard, students build on their knowledge of patterns in the base ten place value system learned in Fourth Grade by reasoning about the magnitude of numbers. Students explain the value of digits in relation to their placement. In addition, students explore the patterns in the placement of digits when multiplied by a power of 10. Students use numerical reasoning to explain the placement of the digits, the values they represent, and by what factor the values have changed (GA DOE, Grade 5 Comprehensive Grade Level Overview).

### Teacher Resource Video:

### Academic Vocabulary:

base, base-ten number system, decimal, decimal point, digit, exponent, hundredth, numerical pattern, place value, power of ten, tenth, thousandth, whole number

NOTE: This list is not intended as a vocabulary list for students, but as a reference for teachers that may be used to ensure precise language is applied and encouraged by all. (GA DOE, Comprehensive Grade Level Overview)

### GADOE Glossary Link

**GCPS Vocabulary Link** 

### Vertical Progressions GA DOE K-12 Learning Progression

Prior to 5th Grade	After 5th Grade	
<ul> <li>Magnitude of place value (4th)</li> <li>Multi-digit whole numbers through the hundred thousands place (4th)</li> <li>Round multi digit whole numbers (4th)</li> <li>Fractions with denominators of 10 or 100 (4th)</li> </ul>	<ul> <li>Students should apply the foundational knowledge of place value to make sense of other mathematical ideas related to numbers in The Real Number System (6th and beyond).</li> </ul>	



Numerical Reasoning			
<ul> <li>Indicator of Achievement</li> <li>1.a explain that in a multi-digit number, a digit in one place represents 10 times as much as it represents in the place to its right (up to 100 times greater) and 1/10 of what it represents in the place to its left (up to 1/1000 of the value (coming soon))</li> </ul>			
IOA Overview			
What does this indicator mean that a student must know, understand, or be able to do? Students work with the idea that the tens place is ten times as much as the ones place, and the ones place is 1/10 the size of the tens place. This standard extends previous understandings of the relationships of the digits in whole numbers only to decimal fractions. Before considering the relationship of decimal fractions, students express their understanding that in multi-digit whole numbers, a digit represents 10 times what it represents in the place to the right and 1/10 of what it represents to the left (GA DOE, Grade 5 Comprehensive Grade Level Overview).			
Instructional Strategies	Considerations		
<b>CONCRETE:</b> Student demonstrates a digit's value as dependent upon its place in the number using base ten blocks or place value disks.	Suggested manipulatives/tools: base ten blocks place value disks <u>powers of 10 place</u> <u>value chart</u> <u>moving digit slider</u>		



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Example	1:

Here are the average weights of:

mouse	.555 lbs
rabbit	5.55 lbs
bulldog	55 lbs
pig	550 lbs

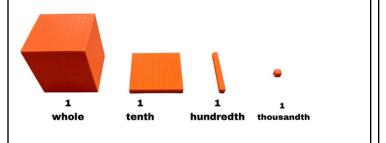
Explicitly teach the concept of exponents during this unit.

Powers of 10, up to 10<sup>3</sup>

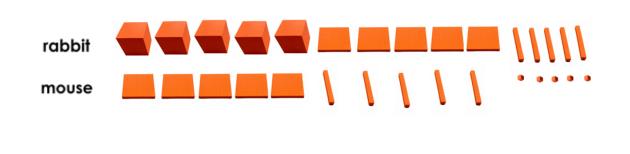
When working with exponents, students must understand that the exponent tells them the number of times the base (in this case 10) is used as a factor.

Directions: Build the weight of the mouse and rabbit using base ten blocks. The large cube equals one whole. Compare the value of the digit 5 in the weights of the mouse and rabbit.

\*Note: To compare the weights of the animals, the whole will need to be redefined. The cube is now the whole, the value of the flat is a tenth, the rod is now a hundredth, and the unit is a thousandth.



#### Sample Student Thinking:



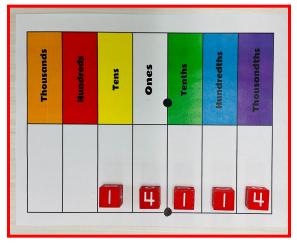


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I know that one cube is 10 times greater than one flat because it takes 10 flats to equal 1 cube. The 5 cubes in the ones place of the rabbit's weight are ten times greater than the 5 flats in the tenths place, so the rabbit is 10 times greater than the mouse.

**REPRESENTATIONAL:** Students use a place value chart to determine each digit's value as compared to another.

**Example:** Jake was creating numbers by rolling dice. He placed them on a place value chart. Here is the number he rolled.



What is the relationship of the digit 1 in the tens place compared to the digit 1 in the tenths place in Jake's number? What is the relationship of the digit 4 in ones place compared to the digit 4 in the thousandths place in Jake's number?

#### Sample Student Thinking:

- The 1 in the tens place is 100 times larger than the 1 in the tenths place.
- The 1 in the tenths place is 1/100 of the value of the 1 in the tens place.



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<b>ABSTRACT:</b> As stu each time a nur which is the bas by 10, the digits			
Example: Comp	pare the weights of the rabl	pit and mouse.	
Here are the av	erage weights of:		
mouse	.55 lbs		
rabbit	5.5 lbs		
bulldog	55 lbs		
pig	550 lbs		
-	-	s place of the mouse's weight is to the right of the 5 in the nouse's weight is $\frac{1}{10}$ of the rabbit's weight.	
		Common Misconceptions	
		oning to decimals and believes the "longer" the number, the In 3 because it is "longer" and has more digits).	greater the value (The student



- Students confuse the language describing the relationship between place values for whole numbers and decimal numbers. For example, when moving from ones to tens (one place to the left), the value is ten times greater, not ten greater or ten more. When moving from tenths to hundredths (one place to the right), the value is 1/10 the value and not ten less.
- Students have difficulty explaining the relationships between the values of the digits in a given number.

Small Group Instructional Moves (1.a & 1.b)



### **Numerical Reasoning**

### Indicator of Achievement

• 1.b: explain patterns in the placement of digits when multiplied or divided by a power of 10; use whole-number exponents to denote powers of 10, up to 10<sup>3</sup>

(via Mix and Math 360)

### IOA Overview

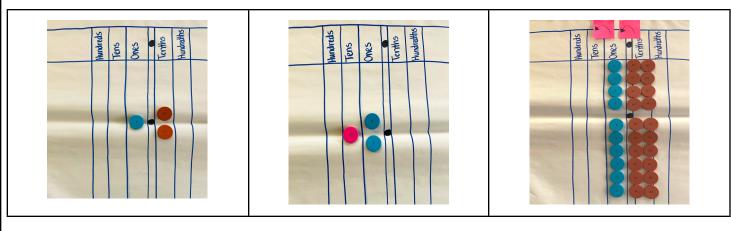
### What does this indicator mean that a student must know, understand, or be able to do?

Students use whole-number exponents to denote powers of 10, up to 10<sup>3</sup>. Students explain what happens to the value of a digit as it shifts to the left or right and discover the decimal point remains between the ones and tenths place as the digits shift. Students reason that the exponent in a power of 10 indicates how many places the digits are shifting. When we multiply by a positive power of 10, the digits shift to the left because the number is increasing by powers of 10. When we divide by a positive power of 10, the digits shift to the right because the number is decreasing by powers of ten. Students need to be provided with opportunities to explore this concept and come to this understanding; this should not be taught procedurally (GA DOE, Grade 5 Comprehensive Grade Level Overview).

Instructional Strategies	Considerations	
<b>CONCRETE:</b> Base ten blocks can be used to model multiplication by powers of 10. Students should understand that the exponent tells how many times the multiplicand is being multiplied by 10.	Suggested manipulatives/tools: • base ten blocks	
<b>Example (Multiplying):</b> Mara has a digital scale. She placed one playing card on the scale and it read 1.2 grams. How much would you expect 10 playing cards to weigh?	<ul> <li>place value disks</li> <li>moving digit slider</li> </ul>	
<b>Sample Student Thinking:</b> A single playing card weighs 1.2 grams. I am trying to figure out how much 10 playing cards weigh. This means I am measuring 10 of an item that weighs 1.2 grams. 10 groups of 1	The decimal point is a convention that has been	



gram weigh 10 grams altogether. 10 groups of 0.2 grams weigh 2 grams altogether. Therefore, 10 playing cards weigh 12 grams. I noticed that when multiplying 1.2 x 10, the digits in 1.2 shifted one place to the left. The value of the 1 and the 2 in 1.2 changed. The 1 shifted to the tens place and now has a value of 10. The 0.2 shifted to the ones place and now has a value of 2. This means that each place is 10 times greater than the place to its right. The decimal point did not move. The digits shifted.



**Example (Dividing):** Chris took the cards off the scale and then placed 10 pennies on the scale and the scale read 24 grams. How much would you expect one penny to weigh?

**Sample Student Thinking:** I know that 10 pennies weigh 24 grams altogether. I am trying to determine how much one penny weighs. If each place on a place value chart is 10 times greater than the place to its right, then I know that each place on a place value chart is 1/10 of what it represents to its left. Therefore, if the total weight of 10 pennies is 24 grams, I can shift the digits in the number 24 one place to the right. The value of the 2 in 24 has shifted from 20 to 2. The value of the 4 in 24 has shifted from 4 to 0.4. This means that 2.4 is 1/10 the value of 24. As the digits shifted, the decimal point did not move. One penny weighs 2.4 grams.

developed to indicate the ones position. The position to the left of the decimal marks the units or ones position, therefore the decimal never moves (<u>Teaching Student</u> <u>Centered Mathematics</u>, Van de Walle et al, 2018).

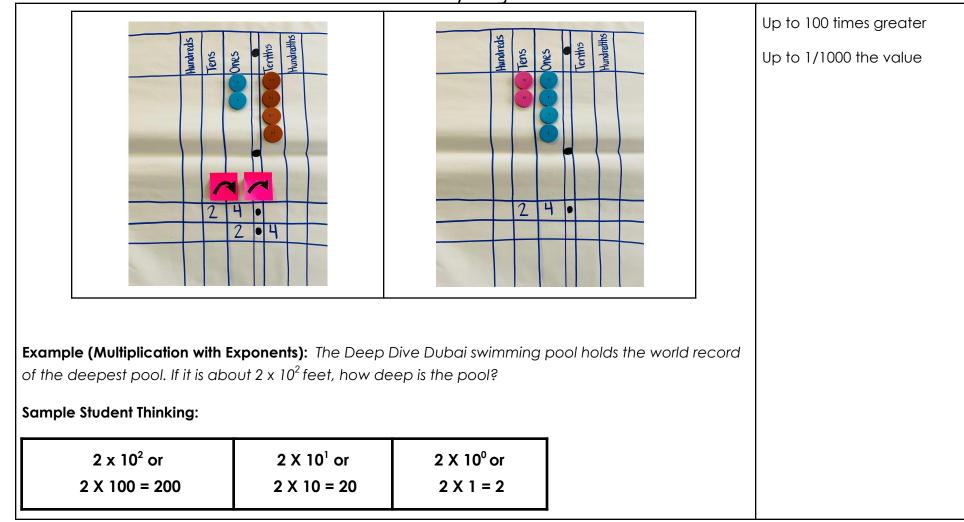
Avoid telling students that the decimal point is moving.

Sentence frames can be given to support students with explaining the relationships between the values of the digits in a given number.

Allow students to make observations and develop their own reasoning behind the patterns prior to doing a think-aloud. This will allow the teacher to provide the students the opportunity to prove/disprove their own theories.

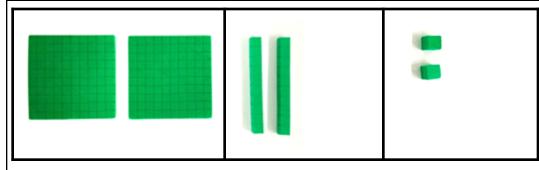


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I know that the Deep Dive Dubai is  $2 \times 10^2$  feet deep.  $10^2$  tells me that I should multiply 10 by itself twice. 10 x 10 is 100, so I can think of  $2 \times 10^2$  as  $2 \times 100$ . I know that 10 times larger than 2 feet is 20 feet. 10 times larger than 20 feet is 200 feet, so the Deep Dive Dubai swimming pool is about 200 feet deep. I can see that the 2 shifted from the ones place to the tens place, and then to the hundreds place for a value of 200.

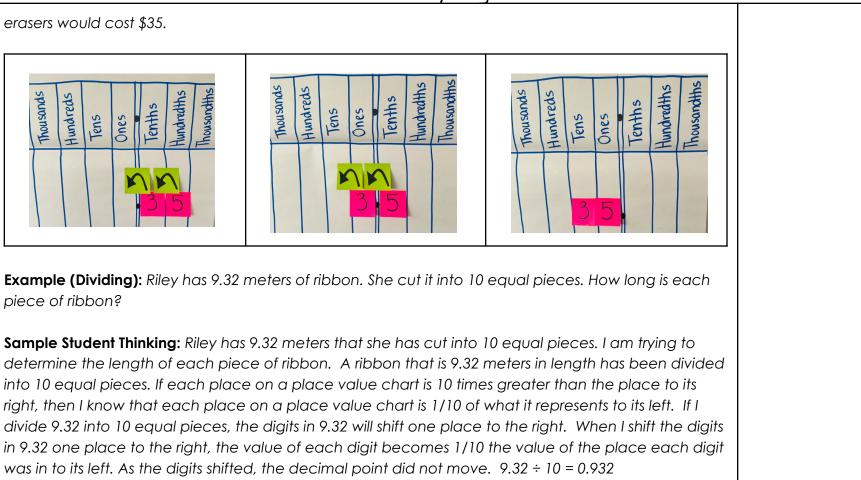
**REPRESENTATIONAL:** Students use patterns that link the exponent to the number of place values the digit is moved as a result of multiplying or dividing by powers of 10. When multiplying, the digits shift to the left in a place value chart. When dividing, the digits shift to the right in a place value chart. A place value chart or interactive place value slider can aid in helping students see these relationships.

Example (Multiplying): An eraser costs \$0.35. How much would it cost to buy 100 erasers?

**Sample Student Thinking:** I know that one eraser costs \$0.35. I am trying to figure out the cost of 100 erasers. I can multiply  $0.35 \times 100$  to determine the total cost of the erasers. I can use a place value chart to show that the value of each place is 10 times greater than the place to its right. I can shift the digits in 0.35 one place to the left to show the cost of 10 erasers. When I make this shift to the left, the value of the 3 and the 5 become 10 tens greater than the place to the right. When I shift the digits in 0.35 one more place to the left, the value of the 3 and the 5 have become 100 times greater than two places to the right. As the digits shifted, the decimal point did not move.  $0.35 \times 100 = 35$ . 100



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**Example (Division with Exponents):** Lauren purchased 10<sup>3</sup> more of a solution than she needed for her experiment. If she has 38 L of the solution, how much does she need for her experiment?

### Sample Student Thinking:

Tens	Ones	•	Tenths	Hundredths	Thousandths
3	8	•	0	0	0
	3	•	8	0	0
	0	•	3	8	0
	0	•	0	3	8

I know that Lauren bought more of the solution than she needed, so that value will be less than 38L. I can use a place value chart to show how the value of each digit shifts. I know that each digit will shift to the right to show 1/10 of the current value. I can use the equation  $38 L \div 10^3 = L$ .  $10^3$  tells me I



should divide each digit by 10 three times which is 1/1000 of 38. 10 times less than 38L is 3 and 8 tenths or 3.8. 10 times less than 3.8 is 38 hundredths or 0.38. 10 times less than .38 is 38 thousandths or .038. So, Lauren will need .038 L of the solution for her experiment. **ABSTRACT:** Students reason that in the base-ten number system, digits to the left are 10 times greater than the digits to the right. Likewise, digits to the right are  $\frac{1}{10}$  of the value of the digits to the left. **Example (Multiplying):** Arti measures the amount of a substance that is formed in a science experiment. Each minute, the substance increases by 10 times the previous amount. If Arti records 0.65 mL after the first minute, how much will she record in 3 minutes? Sample Student Thinking: I am trying to determine the amount Arti's substance increases after 3 minutes. I know that after 1 minute, the substance measured 0.65 mL. I also know that the substance increases by 10 times the previous amount each minute. This means that 0.65 x 10 would tell me the amount of the substance after 2 minutes and 0.65 x 100 would tell me the amount of the substance after 3 minutes. I know that digits to the left on a place value chart are 10 times greater than the digits to the right. Therefore,  $0.65 \times 10 = 6.5$  and  $0.65 \times 100 = 65$ . As the 6 and the 5 shifted to the left one place, their values became 10 times greater than the places to the right. Since the digits in 0.65 moved two places, their value became 100 times greater than where they started. As the digits shifted, the decimal point did not move. **Example (Dividing):** Leroy spent \$136 on 10 movie tickets. How much did he spend on each ticket? Sample Student Thinking: Leroy paid \$136 for 10 movie tickets. I am trying to figure out how much he spent on each ticket. If each place on a place value chart is 10 times greater than the place to its right, then I know that each place on a place value chart is 1/10 of what it represents to its left. I am



value. As the digits shifted, the decimal point did not move. $136 \div 10 = 13.6$ . Leroy spent \$13.60 on each movie ticket.	
dividing 136 into 10 equal parts Therefore, 136 ÷ 10 will tell me the price of one movie ticket. When I shift the digits in 136 one place to the right, the value of each digit becomes 1/10 of its original	

- Students do not understand the meaning of the exponents.
- Students overgeneralize the derived "rule" for multiplying a whole number by a power of 10 to multiplying decimals by powers of 10 without thinking about the value that results (e.g.,  $36 \times 10^2 = 3,600$ , so  $3.6 \times 10^2 = 3.600$ ).
- Students believe that multiplying or dividing by powers of 10 means moving the decimal point to the left and the right.

Small Group Instructional Moves (1.a & 1.b)