Burnette Elementary

Analyzing the Standards

5TH GRADE MATHEMATICS

Standards for Mathematical Practice

5.MP: Display perseverance and patience in problem-solving. Demonstrate skills and strategies needed to succeed in mathematics, including critical thinking, reasoning, and effective collaboration and expression. Seek help and apply feedback. Set and monitor goals.

1 Make sense of problems and persevere in solving them	**2** Reason abstractly and quantitatively	**3** Construct viable arguments/critique the reasoning of others	**4** Model with mathematics
5 Use appropriate tools strategically	**6** Attend to precision	**7** Look for and make use of structure	**8** Look for and express regularity in repeated reasoning

Big Idea: Geometric & Spatial Reasoning

properties of polygons and rectangular prisms, classify polygons

5MA.D.8 examine properties of polygons (e.g., triangles, quadrilaterals including kites, trapezoids, rectangles, squares, rhombuses, other parallelograms, pentagons, hexagons, octagons) and rectangular prisms; classify polygons by their properties; discover volume of right rectangular prisms (5.GSR.8)

- **8.a:** classify, compare, and contrast polygons based on properties
- **8.b:** determine, through exploration and investigation, that attributes belonging to a category of two-dimensional figures also belong to all subcategories of that category
- **8.c:** investigate volume of right rectangular prisms by packing them with unit cubes without gaps or overlaps; determine the total volume to solve problems
- **8.d:** discover and explain how the volume of a right rectangular prism can be found by multiplying the area of the base times the height to solve authentic, mathematical problems
- **8.e:** describe the impact of increasing or decreasing a side length in volume calculations (e.g.; if the height of a prism is

increased by 2 units, what impact does that have on the volume of the rectangular prism?) **(Extension)**
8.f: apply understanding of two-dimensional figures to critique the reasoning of others (e.g., after analyzing an error analysis, plan a list of questions you would ask others to support their understanding and correct the error) **(Extension)**
8.g: create a product, based on an authentic student topic of interest, to demonstrate an understanding of volume concepts **(Extension)**

Overview

What does this standard mean that a student must know, understand, or be able to do?

Description of the Standard

Students classify, compare, and contrast polygons based on properties. Students build on their understanding of shapes by classifying polygons based on their properties. Students will explore geometric properties of shapes to determine relationships between categories and subcategories of shapes. Students can use a variety of manipulatives and real-world objects to build larger shapes to explore the properties and make connections with the key attributes (GA DOE, Grade 5, Unit 7).

Students recognize volume as an attribute of three-dimensional space. They understand that volume can be measured by finding the total number of same size units of volume required to fill the space without gaps or overlaps. They understand that a 1-unit by 1-unit by 1-unit cube is the standard unit for measuring volume. They select appropriate units, strategies, and tools for solving problems that involve estimating and measuring volume. They decompose three dimensional shapes and find volumes of right rectangular prisms by viewing them as decomposed into layers of arrays of cubes. They measure necessary attributes of shapes in order to determine volumes to solve real world and mathematical problems (GA DOE, Grade 5, Unit 1).

Teacher Resource Video: (via Mix and Math 360)

Academic Vocabulary:
angle (right/acute/obtuse), area of base, attribute, category, classify, congruence, cubic units (cubic cm, cubic in, cubic ft, etc.), edge length, equation, expression, gap, height, hexagon, kite, length, liquid volume, octagon, overlap, parallel, parallelogram, pentagon, perpendicular, polygon, property, quadrilateral, rectangle, rhombus, right rectangular prism, solid figure, square,

symmetry, trapezoid, triangle (acute/equilateral/isosceles/scalene), unit cube, volume

NOTE: This list is not intended as a vocabulary list for students, but as a reference for teachers that may be used to ensure precise language is applied and encouraged by all. (GA DOE, Comprehensive Grade Level Overview)

GADOE Glossary Link **GCPS Vocabulary Link**

Vertical Progressions
GA DOE K-12 Learning Progression

Prior to 5th Grade	After 5th Grade
• Explore, investigate, and draw points, lines, line segments, rays, angles (right, acute, obtuse), perpendicular lines, parallel lines, and lines of symmetry. (4th) • Identify points, lines, line segments, rays, angles (right, acute, obtuse), perpendicular lines, parallel lines, and lines of symmetry in two dimensional figures. (4th) • Classify, compare, and contrast polygons based on lines of symmetry, the presence or absence of parallel or perpendicular line segments, or the presence or absence of angles of a specified size and based on side lengths (4th) • Area and perimeter of composite rectangles (4th) • Measure angles in reference to the 360 degrees in a circle with the center at the common endpoint of two rays. (4th)	• Apply geometric and spatial reasoning involving shapes and properties to solve a variety of problems (6th) • Area of triangles, quadrilaterals, and polygons (6th) • Surface area (6th) • Volume of right rectangular prisms with fractional edge lengths (6th)

Burnette Elementary

Analyzing the Standards

Geometric & Spatial Reasoning

Indicator of Achievement

- **8.a:** classify, compare, and contrast polygons based on properties

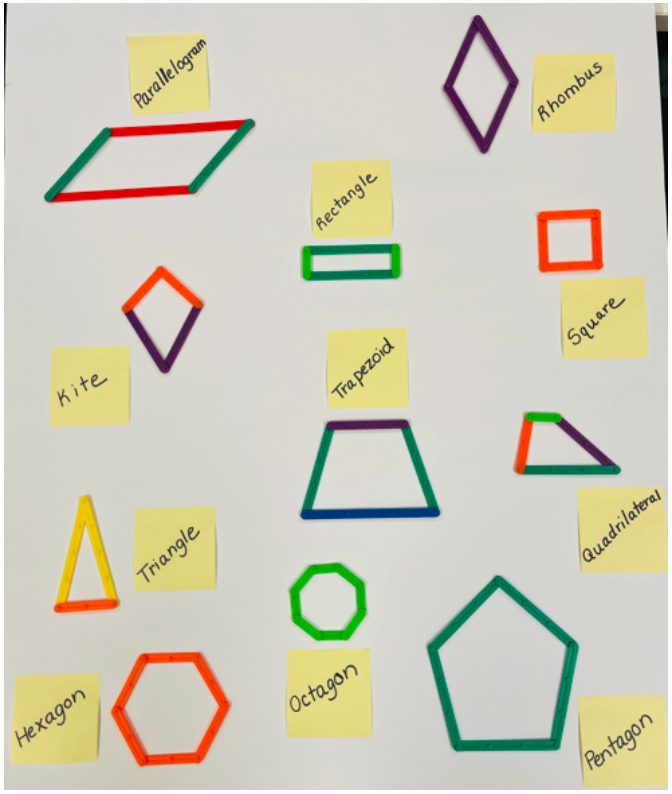
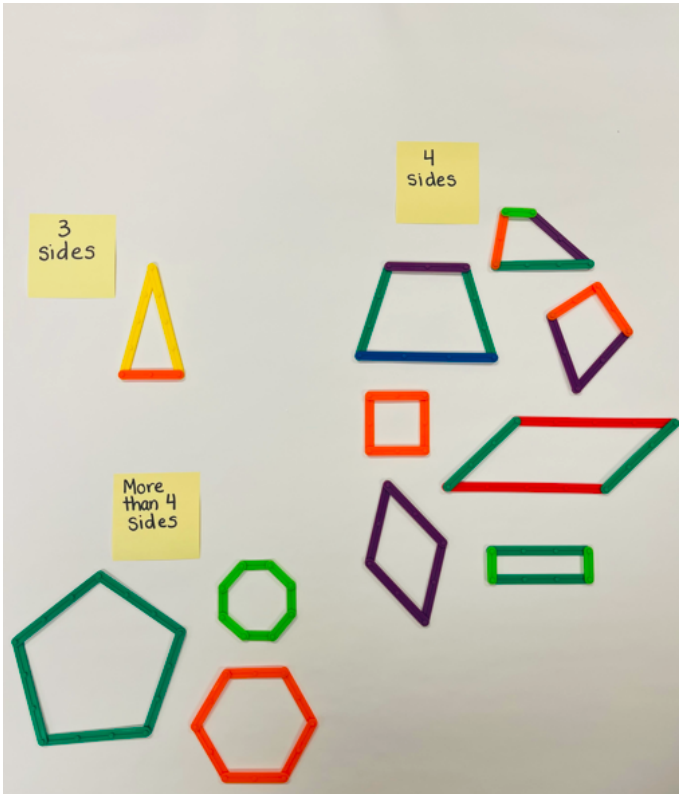
Teacher Resource Video:

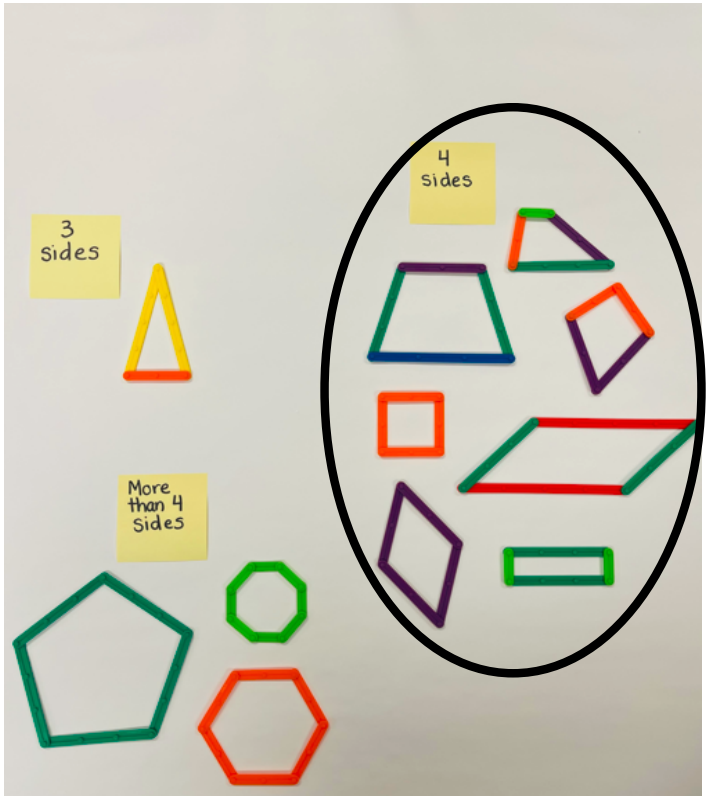
IOA Overview

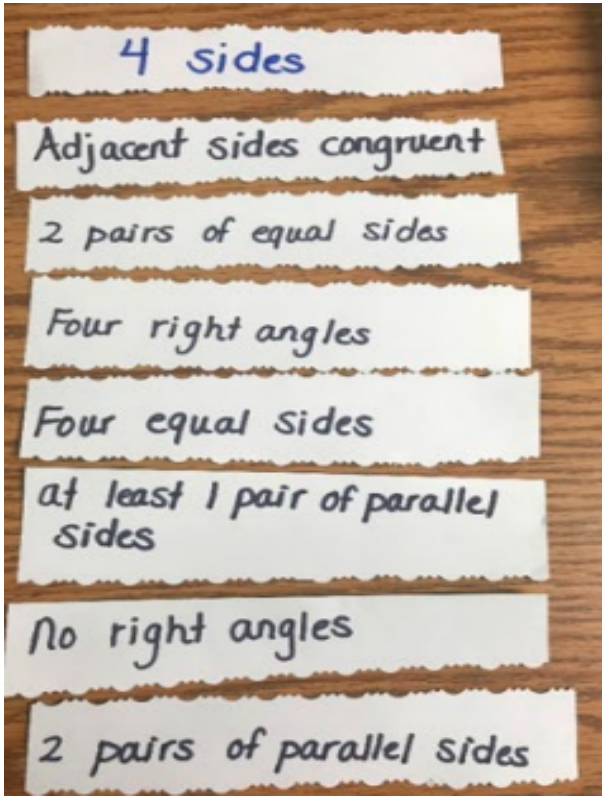
What does this indicator mean that a student must know, understand, or be able to do?

Students classify, compare, and contrast polygons based on properties. Students sort in ways that make sense to them and use attributes to make sense of geometric figures. Polygons include triangles, quadrilaterals including kites and trapezoids (rectangles, squares, rhombuses, and other parallelograms), pentagons, hexagons, and octagons. Properties may include angles, side lengths, symmetry, congruence, and the presence or absence of parallel or perpendicular lines. Students use a variety of tools to measure angles and side lengths to make sense of the properties of polygons. Does not require students to create a hierarchy.
(GA DOE, K-8 Mathematics Standards)

Instructional Strategies	Considerations
CONCRETE: Students understand that two-dimensional shapes are categorized based on their properties (angles, side lengths, symmetry, congruence, and the presence or absence of parallel or perpendicular lines) and that shapes can belong to multiple categories. Students will identify the various categories into which a specific shape may belong. Provide students with opportunities to build shapes using Geostix or geoboards. Allow multiple opportunities for students to compare and contrast shapes based on their attributes. Students may use a variety of tools to measure angles and side lengths to make sense of the properties of polygons.	Suggested manipulatives • Geostix • attribute blocks • pattern blocks • geoboards • tangrams Polygons include triangles, quadrilaterals including kites and trapezoids (rectangles, squares, rhombuses, and

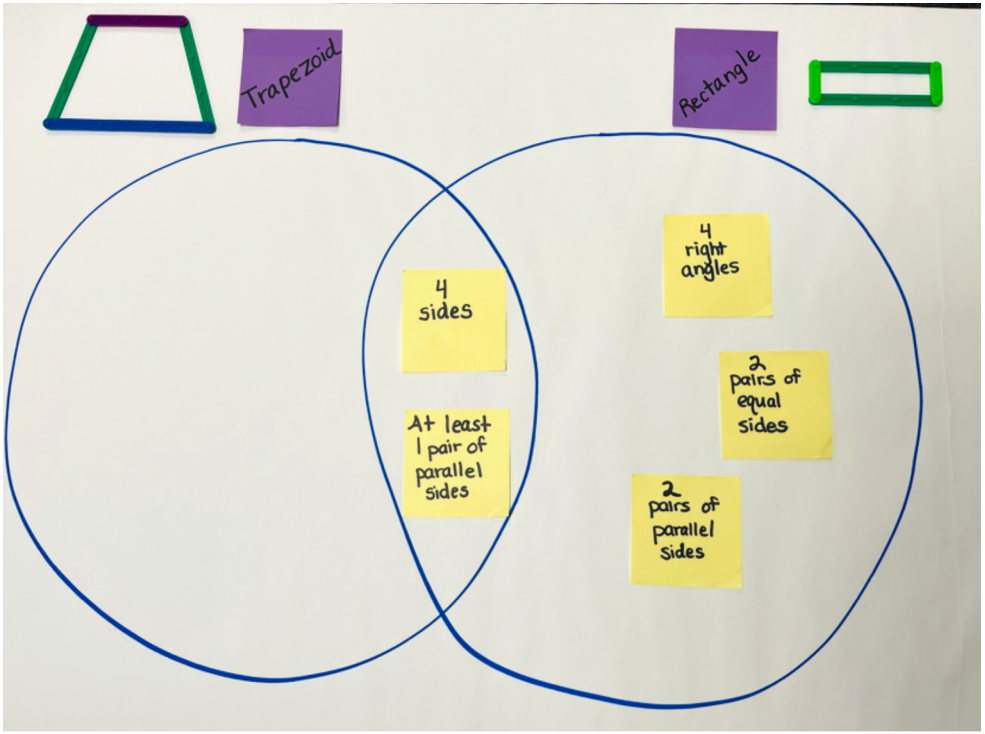
Use Geostix to build different polygons.	Students consider the defining attributes of polygons in order of most general to most specific.	other parallelograms), pentagons, hexagons, and octagons. Properties may include angles, side lengths, symmetry, congruence, and the presence or absence of parallel or perpendicular lines. Does not require students to create a hierarchy Focus on the inclusive definitions for the classification of shapes.
		
Consider a category of polygons.	Determine the attributes for the polygons.	





Note: This is not an exhaustive list of attributes for quadrilaterals.

Students compare and contrast polygons based on attributes.



REPRESENTATIONAL: Decide on an attribute to classify/sort polygons. Students draw polygons that share the given attribute and shapes that do not share the given attribute.

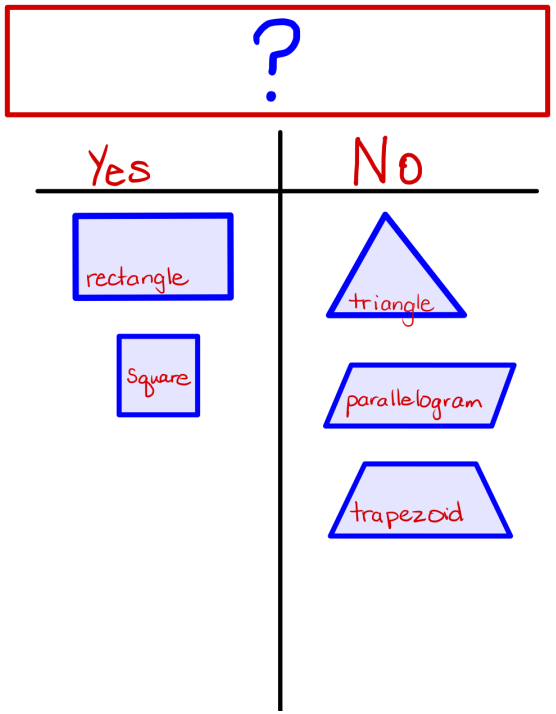
at least 1 pair of parallel sides

Yes	No
rectangle	triangle
rhombus	
Square	
trapezoid	
parallelogram	

- four right angles
- 2 pairs of parallel sides

Yes	No
rectangle	triangle
Square	trapezoid
	parallelogram
	rhombus

Provide students with a given set of shapes and have them determine the attribute.



Sample Student Thinking: *I see the rectangle and the square in the "yes column" and the triangle, parallelogram, and trapezoid are in the "no" column. The rectangle and square have 4 right angles. The triangle, parallelogram, and trapezoid do not have 4 right angles. I think the mystery attribute is "4 right angles".*

ABSTRACT: Students engage in reasoning about polygons and their attributes to consider questions similar to the following (the focus is on attributes, not subcategories):

Example: *Examine whether all quadrilaterals have right angles. Give examples and non-examples.*

Sample Student Thinking: *I know that a rectangle, square, parallelogram, trapezoid, and rhombus are all quadrilaterals. A rectangle and a square have right angles (examples). A trapezoid, rhombus, and*

parallelogram do not have any right angles (non-examples). Therefore, all quadrilaterals do not have right angles.	

Common Misconceptions

- Students are unable to classify shapes according to multiple attributes.

Burnette Elementary
Analyzing the Standards

Geometric & Spatial Reasoning
Indicator of Achievement • **8.b:** determine, through exploration and investigation, that attributes belonging to a category of two-dimensional figures also belong to all subcategories of that category **Teacher Resource Video:** (coming soon)
IOA Overview
What does this indicator mean that a student must know, understand, or be able to do? Students explore geometric properties of shapes to determine relationships between categories and subcategories of shapes. Students can use a variety of manipulatives and real-world objects to build larger shapes to explore the properties and make connections with the key attributes (GA DOE, Grade 5 Comprehensive Grade Level Overview).

Instructional Strategies	Considerations
CONCRETE: Students build shapes to compare shapes belonging to a category of two-dimensional figures to make comparison statements. **Example:** *ALL PARALLELOGRAMS ARE RECTANGLES* *Lisa said the statement is true, but Brian thinks all rectangles are parallelograms. Who do you agree with and why?* Students build each quadrilateral using concrete manipulatives like Geostix or geoboards to explore properties and determine relationships between categories and subcategories of shapes.	Suggested manipulatives: • Geostix • attribute blocks • pattern blocks • geoboards • tangrams Does not require students to create a hierarchy Focus on the inclusive definitions for the

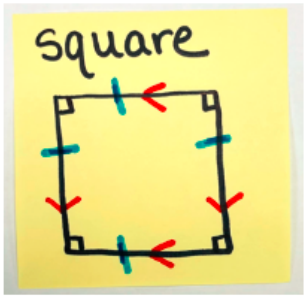
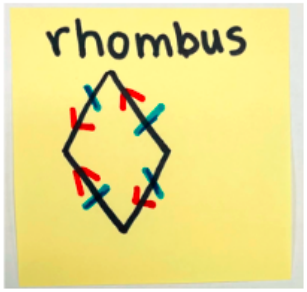
2D Shape	Attributes
	• 2 pairs of parallel sides • Opposite sides are congruent • Opposite angles are congruent • 4 right angles
	• 2 pairs of parallel sides • Opposite sides are congruent • Opposite angles are congruent

Sample Student Thinking: *The rectangle and the parallelogram share some of the same attributes. Both shapes have 2 pairs of parallel sides. They also have opposite sides that are congruent and opposite angles that are congruent. Brian is correct, because rectangles have all of the attributes of a parallelogram. Therefore, all rectangles are parallelograms. A parallelogram is not a rectangle, because it does not share the attribute of having 4 right angles. Therefore, Lisa is not correct.*

classification of shapes.

REPRESENTATIONAL: Students build shapes to compare shapes belonging to a category of two-dimensional figures to make comparison statements.

Example: *ALL SQUARES ARE RHOMBI.*
Sarah said the statement is true. Susan, however, said the statement is not true. Who do you agree with and why?

2D Shape	Attributes
	• 2 pairs of parallel sides • 4 equal sides • 4 right angles
	• 2 pairs of parallel sides • 4 equal sides • Some rhombi have 4 right angles (this one does not)

Sample Student Thinking: *The statement says that "all squares are rhombi". This means that every square would have to have the same attributes as a rhombus for this to be true. I agree with Sarah that all squares are rhombi. Rhombi have 4 sides that are equal in length and two pairs (or sets) of*

parallel sides. Squares also share these attributes. Therefore, all squares are rhombi. The opposite would not be true, because some rhombi have 4 right angles (depending on the rhombus) but not all.	
ABSTRACT: Students engage in reasoning about polygons and their attributes to consider subcategories within the same category through questions similar to the following: **Example:** *A trapezoid has a pair of parallel sides therefore, it must be a parallelogram. Support or dispute.* **Sample Student Thinking:** *Before I can place a trapezoid in the subcategory of "parallelograms", I know that a trapezoid must have all of the attributes of a parallelogram. A trapezoid must have at least 1 pair of parallel sides. A parallelogram must have 2 pairs of parallel sides. A trapezoid does not have this attribute, so it cannot be a parallelogram.*	

Common Misconceptions

- Students are unsure of definitions that involve inclusive relations of quadrilaterals
- Students think that when describing geometric shapes and placing them in subcategories, the last category is the only classification that can be used.

Burnette Elementary
Analyzing the Standards

Geometric & Spatial Reasoning

Indicator of Achievement

- **8.c:** investigate volume of right rectangular prisms by packing them with unit cubes without gaps or overlaps; determine the total volume to solve problems

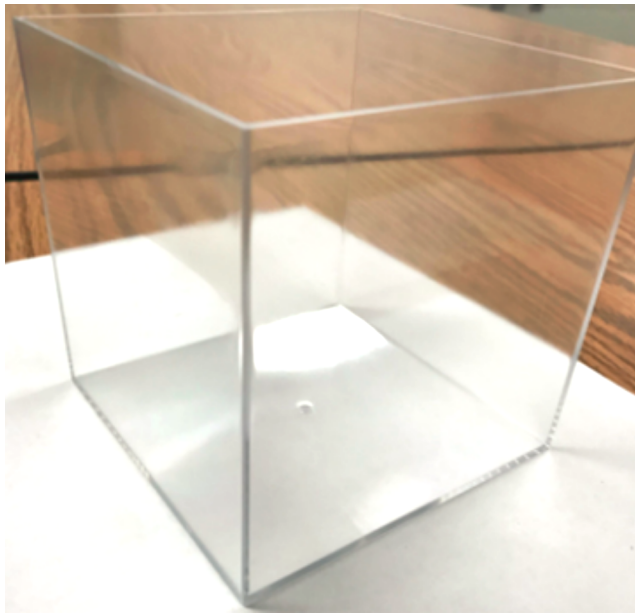
Teacher Resource Video: (coming soon)

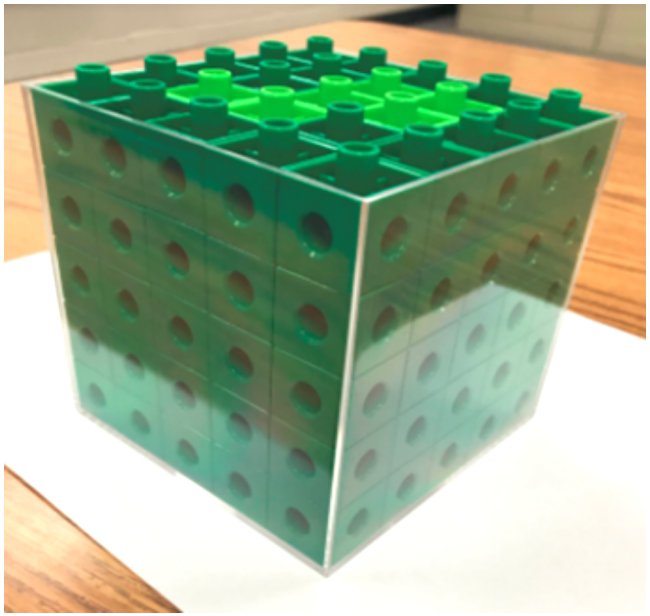
IOA Overview

What does this indicator mean that a student must know, understand, or be able to do?

Students recognize volume as an attribute of solid figures. Students find the volume of right rectangular prisms and extend previous understandings of finding the area of composite figures into the context of volume. Students explore the volume of solid figures from realistic situations by packing them with no gaps or overlaps. Students determine that a solid figure packed with *n* unit cubes is said to have a volume of *n* cubic units. Students investigate authentic problems involving volume to make sense of this concept (GA DOE, Grade 5 Comprehensive Grade Level Overview).

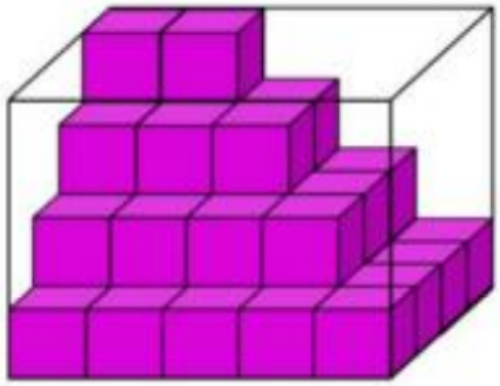
Instructional Strategies	Considerations
CONCRETE: Provide students with a variety of rectangular prisms (boxes such as paper clip box, cassette case, small game box, pizza box, 12-pack soda box, copy paper box, etc.) and appropriate cubes as standards of measure (cubes from base ten kit, snap cubes). Provide time for students to fill containers with provided units. Students should recognize that volume is determined by filling a container with unit cubes so there are no gaps or overlapping units. **Example:** *Mr. Sinyard was trying to fill a container with cubes. There are no gaps or overlapping units. It took 125 cubes to fill the container. Therefore, the figure has a volume of 125 cubic units.*	Suggested manipulatives • Unifix/snap cubes • centimeter cubes • base ten blocks If students are provided with an image of a right rectangular prism, the unit cubes should be visible.





REPRESENTATIONAL: Students analyze images of right rectangular prisms with visible unit cubes to solve authentic problems involving volume.

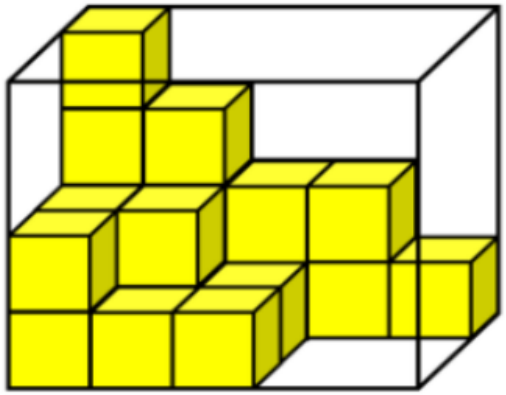
Example 1: *Evan was filling a container with cubic blocks. How many cubes can be added to this container without overlapping or overflowing? How do you know?*



Sample Student Thinking: *I can see that the bottom layer, or base is 5 rows with 4 cubic units in each row. 5 + 5 + 5 + 5 = 20. I could also multiply 5 x 4 to get 20. So, I know that the bottom layer has 20 cubic units. I can see that the container has 4 layers of blocks. If each layer has 20 blocks, then I can add 20 + 20 + 20 + 20 or I can multiply 20 x 4 to determine that 80 cubic units are needed to fill the box. The total volume of the container is 80 cubic units.*

The second layer has 12 cubes. The third layer has 6 cubes. The top layer has 2 cubes. 12 + 6 + 2 = 20 cubic units. If I add the 20 cubic units from the bottom layer with the 20 cubic units shown in the other layers, I would have a total of 40 cubic units. I am trying to find how many cubes can be added to the container without overlapping or overflowing. 80 - 40 = 40 cubic units. Evan can add 40 cubic units to the container.

Example 2: *Andrew has been filling a box with cubes. If he fills the rest of this box with cubes, what would be the volume of this prism?*



Sample Student Thinking: *I can see that the bottom layer of the box has a length that is 5 cubes long from end to end. I can also see the width is 3 cubic units wide. I can add 5 + 5 + 5, or I could multiply 5 x 3 to determine the bottom layer has 15 cubic units.*

I can also see that the box can fit 4 layers of cubes with no gaps or overlaps and without overflowing. If one layer has 15 cubic units, then I can add 15 + 15 + 15 + 15, or I can multiply 15 x 4 to determine the total volume of the box. 15 + 15 + 15 + 15 = 60. The volume of the box is 60 cubic units.

ABSTRACT: The abstract level for this IOA presents itself in 5.GSR.8.4d.

Common Misconceptions

- Students struggle to understand the concept of volume (packing without gaps or overlaps).
- Students struggle to determine the total volume of a right rectangular prism.

Small Group Instructional Moves (8.c & 8.d)

Geometric & Spatial Reasoning

Indicator of Achievement

- **8.d:** discover and explain how the volume of a right rectangular prism can be found by multiplying the area of the base times the height to solve authentic, mathematical problems

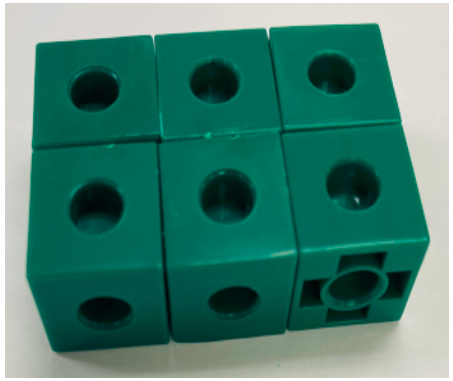
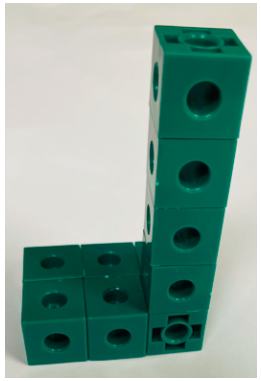
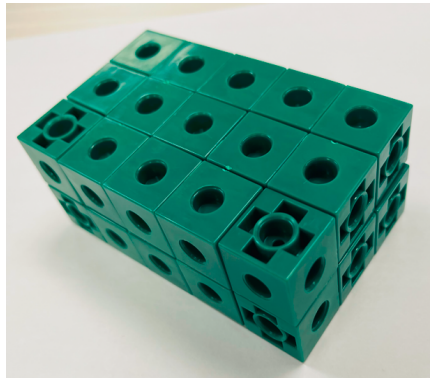
Teacher Resource Video:

IOA Overview

What does this indicator mean that a student must know, understand, or be able to do?

Students describe and reason how the volume of a right rectangular prism can be found by multiplying the area of the base times the height is true. Specifically, students cover the bottom of a right rectangular prism (length x width) with multiple layers (height). Therefore, the formula (length x width x height) is an extension of the formula for the area of a rectangle. The focus of this expectation is for students to understand the concept of volume rather than the formula. Students investigate authentic problems involving volume to make sense of this concept (GA DOE, Grade 5 Comprehensive Grade Level Overview).

Instructional Strategies	Considerations
CONCRETE: Students use concrete manipulatives like snap cubes to discover and explain how to find the volume of a right rectangular prism by multiplying the area of the base times the height. **Example:** *Mr. Adams bought a freezer. The freezer is rectangular and the space inside it measures 3 feet long by 2 feet wide by 5 feet high. What is the volume of the space inside the freezer?*	Suggested manipulatives: • Unifix/snap cubes • centimeter cubes • base ten blocks Explore the dimensions of all possible rectangular prisms given a total number of cubic units.

Sample Student Thinking: *I used snap cubes to build a model of the freezer. It is 3 cubes long, 2 cubes wide, and 5 cubes high. First, I built the base layer which is 3 cubes long and 2 cubes wide. I can multiply 3 x 2 to find the area of the base. There are 6 cubic units in the base. Then, I can build up the height which is 5 cubes high. I already have 1 layer, so I only need to build 4 more layers to have 5. Each layer will be 3 x 2 and have 6 cubic units. I can also take the area of the base which was 3 x 2 and multiply it by 5, the height to find the volume of the space inside the freezer.*

(3 x 2) x 5 = 30

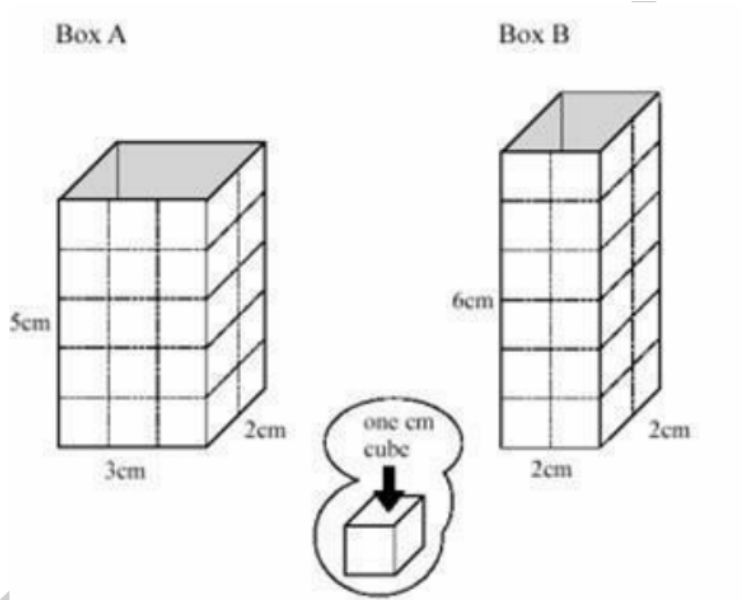
It took 30 cubes to fill the freezer model. Therefore, the volume of the space inside the freezer is 30 cubic feet, or 30 ft^3.

REPRESENTATIONAL: Students will realize that the volume of a box can be determined by three dimensions. The area of the base can be determined by its length and width, and the number of layers can be determined by finding the height of the box, so the three dimensions that determine volume include length, width, and height. The volume can be expressed as the product of the three dimensions using the associative property of multiplication.

Students should have experiences to describe and reason about why the formula is true as the focus is on the concept of volume rather than the formula itself.

Students will be able to look at a drawing of a rectangular prism which has cubic units drawn on it and count the number of snap cubes along the length and width of the prism's base as well as the number of cubes along its height/the number of layers. Students will be able to use those dimensions to determine the prism's volume.

Example 1: *Elliot fills Box A and Box B with one-centimeter cubes. How many cubes can fit into Box A? How many cubes can fit into Box B? Which of the two boxes can hold more cubes? Explain how you figured it out.*



Sample Student Thinking: *Box A has a length of 3 cm, a width of 2 cm, and a height of 5 cm. I am trying to determine how many cubes can fit into Box A. I can find the area of the base layer by multiplying the length (3) by the width (2). Then, I can multiply the area of the base by the height (5)*

to find the volume. This will tell me how many cubes can fit into Box A.

(3 x 2) x 5 = 30 cm^3

Box A can hold 30 cubes.

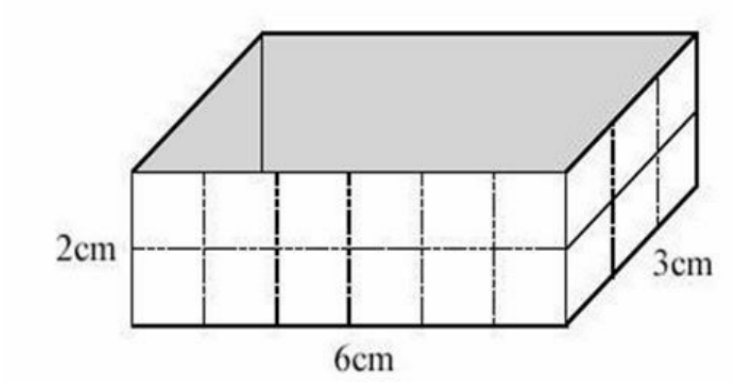
Box B has a length of 2 cm, a width of 2 cm, and a height of 6 cm. I am trying to determine how many cubes can fit into Box B. I can find the area of the base layer by multiplying the length (2) by the width (2). Then, I can multiply the area of the base by the height (6) to find the volume. This will tell me how many cubes can fit into Box B.

(2 x 2) x 6 = 24 cm^3

Box B can hold 24 cubes.

Box A can hold 30 cubes and Box B can hold 24 cubes. Therefore, Box A has a greater volume and can hold more cubes.

Example 2: *A box has the following dimensions:*



How many centimeter cubes can this box hold? Find the dimensions of another box that can hold the

same amount as this box.

_____ cm long _____ cm wide _____ cm high

Sample Student Thinking: *The box has a length of 6 cm, a width of 3 cm, and a height of 2 cm. I am trying to find out how many centimeter cubes the box can hold. I can find the area of the base by multiplying the length (6) by the width (3). I can then multiply the area of the base by the height (2) to determine the total volume, which will tell me how many centimeter cubes the box can hold.*

(6 x 3) x 2 = 36 cm^3

The box can hold 36 cubes.

I am also trying to find the dimensions of another box that can also hold 36 cubes. I could have a box with a length of 9 cm and a width of 2 cm. I know that 9 x 2 = 18. If the area of the base has 18 cubes, I can multiply it by the height to get 36.

(9 x 2) x 2 = 36 cm^3

ABSTRACT: Students use the information in a real world problem to decide which volume formula to use and apply that formula to answer the question.

Example 1: *Melissa has a cube that measures 5 cm on each side. What is the volume of the cube?*

Sample Student Thinking:

Option 1: I know that I can find volume by multiplying the area of the base times the height. If Melissa's figure is a cube, that means it measures 5 cm on each side. That means the length is 5 cm, the width is 5 cm and the height is 5 cm.

(5 x 5) x 5 = 125

Option 2: I know I can find the volume of a right rectangular prism by multiplying the area of the base (B) by the height (h), the formula for volume.

V = B x h

B = 25 h = 5

25 x 5 = 125.

Therefore, the volume of Melissa's cube is 125 cm^3.

Example 2: *Juan wants to figure out how much space there is in his truck bed. The bottom of the truck bed covers 24 square feet, and it is 3 feet high. How much space is there in the bed of his truck?*

Sample Student Thinking: *I know that the bottom of the truck bed covers 24 square feet, and the truck has a height of 3 feet. To find the volume of the truck bed, I can use the volume formula V = B x h where the B is the area of the base and the h is the height.*

B = 24 h = 3

24 x 3 = 72

Therefore, there are 72 square feet in the bed of the truck.

Common Misconceptions

- Students have difficulty describing what area is in order to use the Area of the Base x height and believe they are adding the dimensions
- Students have difficulty reasoning about how the area of the base relates to volume

Small Group Instructional Moves (8.c & 8.d)

Geometric & Spatial Reasoning

Indicator of Achievement

- **8.e:** describe the impact of increasing or decreasing a side length in volume calculations (e.g.; if the height of a prism is increased by 2 units, what impact does that have on the volume of the rectangular prism?) **(Extension)**

IOA Overview

What does this indicator mean that a student must know, understand, or be able to do?

The purpose of this extension IOA is for students to use their creativity, critical thinking, problem-solving, and reasoning skills to determine their own way to apply the understanding of this AKS through various platforms to show the application of skills and concepts in the real world.

Allow students the opportunity to investigate building rectangular prisms that the side length increases or decreases by building prisms with manipulatives or online programs. Have students create a chart to record findings and determine patterns and justify their reasoning.

Geometric & Spatial Reasoning

Indicator of Achievement

- **8.f:** apply understanding of two-dimensional figures to critique the reasoning of others (e.g., after analyzing an error analysis, plan a list of questions you would ask others to support their understanding and correct the error) **(Extension)**

IOA Overview

What does this indicator mean that a student must know, understand, or be able to do?

This extension IOA allows for the natural integration of Math Practices #3 which states that students must construct viable arguments and critique the reasoning of others. Guiding questions to evaluate the students' performance may be used to reflect on students' critique of classmates' reasoning after analysis of the scenario. Students should also be able to identify a list of questions to ask peers.

Geometric & Spatial Reasoning

Indicator of Achievement

- **8.g:** create a product, based on an authentic student topic of interest, to demonstrate an understanding of volume concepts **(Extension)**

IOA Overview

What does this indicator mean that a student must know, understand, or be able to do?

The purpose of this extension IOA is for students to use their creativity, critical thinking, problem-solving, and reasoning skills to determine their own way to apply the understanding of this AKS through various platforms to show the application of skills and concepts in the real world.

Allow students to determine how they will identify and create a product that demonstrates their understanding of volume. Students may research to determine how volume is used in the real-world to solve a problem.

Suggestions, although not exhaustive:

Students are able to create a product (item which is used by others) or a product (type of demonstration) through art, technology (video/animation, etc), commercial, graphic design, and oral reports with the collection of pictures.